## VI. Construction of the Regular Heptagon by a Quartic Curve.

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Consider the quartic

$$x(a+y^2)\sqrt{-(b^2+a^7)} = a^3y^4 - by^3 + 4a^4y^2 - 3aby + 2a^5, \tag{1}$$

which will be real when $b^2 + a^7$ is negative, and the circle

$$x^2 + y^2 = -4a, \tag{2}$$

which will be real when $a$ is negative.

Eliminating $x$ from (1) and (2), the result may be placed under the form

$$(y^7 + 7ay^5 + 14a^2y^3 + 7a^3y + 2b)(a^3y - 2b) = 0.$$

By the transformation $y = -2s\sqrt{-a}$ the first of these factors when placed equal to zero may be reduced to

$$7s - 56s^3 + 112s^5 - 64s^7 = \frac{b}{a^3\sqrt{-a}};$$

and because

$$7\sin A - 56\sin^3 A + 112\sin^5 A - 64\sin^7 A = \sin 7A,$$

we may take

$$s = \sin A, \qquad \frac{b}{a^3\sqrt{-a}} = \sin 7A$$

and get

$$y = -2\sqrt{-a}\sin\frac{1}{7}\sin^{-1}\frac{b}{a^3\sqrt{-a}},$$

$$y = -2\sqrt{-a}\sin\frac{1}{7}\left(2\pi + \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right),$$

$$y = -2\sqrt{-a}\sin\frac{1}{7}\left(3\pi - \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right),$$

$$y = -2\sqrt{-a}\sin\frac{1}{7}\left(\pi - \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right),$$

$$y = 2\sqrt{-a}\sin\frac{1}{7}\left(\pi + \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right),$$

$$y = 2\sqrt{-a}\sin\frac{1}{7}\left(3\pi + \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right),$$

$$y = 2\sqrt{-a}\sin\frac{1}{7}\left(2\pi - \sin^{-1}\frac{b}{a^3\sqrt{-a}}\right).$$

The seven intersections, whose ordinates are the seven real roots of the above-mentioned factor, are the vertices of the regular heptagon required, as may be seen by reference to the above values of $y$.

The above factor of the seventh degree may be resolved algebraically by the transformation

$$y = x - \frac{a}{x}.$$

The result is

$$y = (-b + \sqrt{b^2 + a^7})^{1/7} + (-b - \sqrt{b^2 + a^7})^{1/7}.$$

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# RECENT PUBLICATIONS.

## REVIEWS.

*Geschichte der Mathematik. II Teil, von Cartesius bis zur Wende des 18. Jahrhunderts; II. Hälfte, Geometrie und Trigonometrie.* By HEINRICH WIELEITNER. Berlin, 1921, pp. vi + 220. Price, in Germany, 45 marks.

This small volume, one of the latest numbers in the well-known Sammlung Schubert, completes a work begun thirteen years ago by Professors Günther and Braunmühl,—a work much delayed by the World War. Like all volumes appearing in the Schubert series, it aims at presenting the best attainable knowledge, in a somewhat popular style, by a scholar of recognized standing, and with such a condensation of material as shall allow for its publication at a price within the reach of every teacher in Germany.

In the present work, Dr. Wieleitner has divided his material into nine chapters, as follows: I. Analytic geometry of the plane, especially as related to conic sections; II. Analytic geometry of space, including a study of surfaces; III. Higher curves in general; IV. Special curves; V. Differential geometry; VI. Perspective, and Descriptive geometry; VII. The first steps in projective geometry; VIII. Trigonometry; IX. Elementary geometry.

This is a wide range of subjects to be treated with any thoroughness in 176 pages of text, and yet it is safe to say that Dr. Wieleitner, as might have been expected from one of his scholarship and experience as a writer, has kept up the best traditions of the Sammlung Schubert. For example, he has presented in only fourteen pages the essential features of those contributions to the invention of analytic geometry made by Fermat and, to a lesser extent, by Vieta, Ghetaldi, and Cataldi, as well as those appearing in the epoch-making work of Descartes himself. In the following nineteen pages he condenses those essential topics in analytic geometry which attracted the attention of the contemporaries and the immediate successors of Descartes. Not only are the prominent features set forth, but the student is furnished with a helpful bibliography that allows him to branch out for himself,—a contribution that cannot be too highly commended.

Among the notable features of the work should be mentioned the list of curves in chapter IV. This is, of course, not so complete as the one given by Brocard in his *Notes de Bibliographie des Courbes Géométriques*, but it covers the important curves that the college student will meet in his studies, and includes a list of bibliographical references that will be of much service.